Natural Convection Due to Solar Radiation over a Non-Absorbing Plate with and without Heat Losses

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Natural convection over a non-reflecting, non-absorbing, ideally transparent semi-infinite vertical flat plate due to absorption of incident radiation (solar radiation) is considered. The absorbed radiation acts as a distributed source which initiates buoyancy-driven flow and convection in the absorbing layer. The plate when heated by the absorbing fluid loses heat to the surroundings from its external side. Solution of the governing equations of the flow under these circumstances is non-similar because of both the heat source term in the energy equation and the temperature boundary condition at the plate. A local non-similar technique is used to obtain solutions for a wide range of the dimensionless distance along the plate and of the dimensionless loss coefficient to the surroundings. The results show that the temperature distribution has a maximum temperature in the depth of the fluid rather than on the plate. A new definition for a local heat transfer coefficient between the plate and the absorbing fluid is introduced which is based on the local maximum temperature rise in the fluid. A formula to calculate this heat transfer coefficient is given for the anticipated range of the loss coefficient.

NOTATION

- a absorption coefficient of the fluid, m^{-1}
- A, B constants appearing in eq. (20)
- c specific heat of the fluid, J/kg K
- F dimensionless stream function, defined by eq. (6)
- g gravitational acceleration, m/s^2
- G derivative of the dimensionless velocity with respect to ζ, $G = \partial F/\partial\zeta$
- G_x local Grashof number, defined by eq. (5)
- G_a Grashof number based on the absorption coefficient of the fluid, defined by eq. (5)
- h heat transfer coefficient to the absorbing fluid, W/m^2 K
- k thermal conductivity of the absorbing fluid, W/m K
- N_c dimensionless loss coefficient to the surroundings, defined by eq. (10)
- Nu_a Nusselt number based on the absorption coefficient of the fluid, defined by eq. (19)
- q'' incident radiation flux, W/m^2
- T local temperature, K
- T_p plate temperature, K
- T_∞ temperature of surroundings, and also of the absorbing fluid far from the plate
- U heat transfer coefficient to surroundings, W/m^2 K
- u longitudinal velocity, m/s
- v normal velocity, m/s
- x distance along the plate, m
- y distance normal to the plate, m
- β coefficient of volumetric expansion, K^{-1}
- ζ dimensionless distance along the plate, defined by eq. (4)
- η dimensionless distance normal to the plate, defined by eq. (4)
- θ dimensionless temperature, defined by eq. (6)
- ν kinematic viscosity of fluid, m^2/s
- ρ density of fluid, kg/m^3
- ϕ derivative of the dimensionless temperature with respect to ζ, $\phi = \partial\theta/\partial\zeta$
- ψ stream function, m^2/s

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INTRODUCTION

Buoyancy-driven flows over flat plates have received attention from many investigators. Isothermal vertical plates have been considered by Schmidt and Beckmann (**1**), Pohlhausen (**2**), Ostrach (**3**), and Yang and Jerger (**4**). Kierkus (**5**), Hassan and Mohamed (**6**), and Elsayed (**7**) considered the inclined plates. The uniform heat flux case was considered by Sparrow and Gregg (**8**) among others. The present work deals with natural convection over a non-reflecting, non-absorbing, ideally transparent vertical flat plate due to absorption of incident radiation in the adjacent fluid. Being heated by the absorbing fluid, the plate loses heat to surroundings on its external side. The same configuration was studied by Elsayed and Fathalah (**9**) for forced convection. This problem has wide applications in solar collectors with direct solar collection using an absorbing fluid.

MATHEMATICAL MODEL

The model considered is shown schematically in Fig. 1. The non-reflecting non-absorbing ideally transparent semi-infinite vertical flat plate receives an incident radiation flux of intensity q''. This radiation flux penetrates the plate and is absorbed in an adjacent fluid of absorption coefficient a. The plate loses heat to the surroundings with heat transfer coefficient U. The fluid is assumed to be isotropic, Newtonian, and with all properties taken as constant except the density in the buoyancy term (Boussinesq approximation). In the cartesian coordinate system shown in Fig. 1, the governing equations for steady incompressible flow in terms of the

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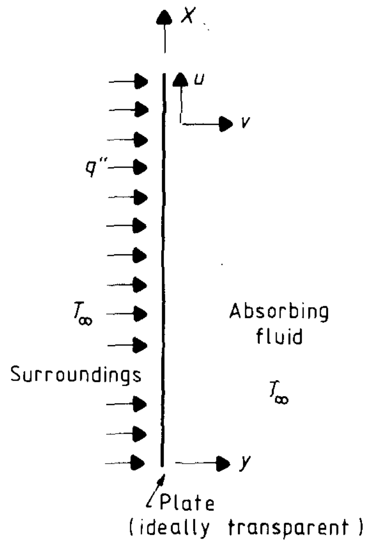

Fig. 1. Sketch of the present model

stream function ψ and the fluid temperature T are

$$\psi_y\psi_{xy} - \psi_x\psi_{yy} = \nu\psi_{yyy} + g\beta(T - T_\infty) \quad (1)$$

$$\psi_y T_x - \psi_x T_y = \frac{k}{c\rho} T_{yy} + \frac{aq''}{c\rho} e^{-ay} \quad (2)$$

where T_∞ is the temperature of both the surroundings and the absorbing fluid at a far distance from the plate. The subscripts x and y in the above equations indicate differentiations with respect to x and y, respectively. The boundary conditions for an infinitely thin plate are as follows

$$\begin{aligned} &\text{at } y = 0, \psi_y = 0, \quad \psi_x = 0, \quad kT_y = U(T - T_\infty) \\ &\text{at } y = \infty, \psi_y = 0, \quad T = T_\infty \end{aligned} \quad (3)$$

In order to solve the above governing equations, a transformation from the (x, y) plane to the (ζ, η) plane is considered. The relations between the (ζ, η) coordinates and the (x, y) coordinates are

$$\zeta = G_x/G_a^5, \qquad \eta = \frac{y}{5x} G_x \quad (4)$$

where G_x and G_a are modified Grashof numbers based on the distance 'x' and the absorption coefficient 'a', respectively. The definitions of G_x and G_a are given by

$$G_x = 5\left(\frac{g\beta q'' x^4}{5k\nu^2}\right)^{1/5}, \qquad G_a = 5\left(\frac{g\beta q''}{5k\nu^2 a^4}\right)^{1/5} \quad (5)$$

Similarly, the dependent variables ψ and T are replaced by a dimensionless stream function F and a dimensionless temperature θ given by

$$F(\zeta, \eta) = \frac{\psi}{\nu G_x}, \qquad \theta(\zeta, \eta) = \frac{k}{5xq''} G_x(T_\infty - T) \quad (6)$$

Substitution of eqs. (4)–(6) into eqs. (1) and (2) gives the following dimensionless governing equations

$$F''' + 4FF'' - 3F'^2 - \theta = -4\zeta[F''G - F'G'] \quad (7)$$

$$\theta + Pr(4F\theta' - F'\theta) = 4\zeta Pr[F'\phi - \theta' G] + 5\zeta^{1/4} \exp(-5\zeta^{1/4}\eta) \quad (8)$$

where Pr is Prandtl number, $G = \partial F/\partial\zeta$, $\phi = \partial\theta/\partial\zeta$, and the primes indicate differentiation with respect to η. The boundary conditions for eqs. (7) and (8) are obtained from eq. (3) after transformation to the (ζ, η) plane, thus

$$\begin{aligned} &F(\zeta, 0) = F'(\zeta, 0) = \theta'(\zeta, 0) - 5N_c\zeta^{1/4}\theta(\zeta, 0) = 0 \\ &F'(\zeta, \infty) = \theta(\zeta, \infty) = 0 \end{aligned} \quad (9)$$

where N_c is the loss coefficient, defined by

$$N_c = U/ka \quad (10)$$

In the local non-similar technique used by Sparrow and Yu (**10**), an additional set of equations that governs G and ϕ is obtained by differentiating eqs. (7)–(9) with respect to ζ. The derivatives $\partial G/\partial\zeta$ and $\partial\phi/\partial\zeta$ are neglected for the second level of truncation. These additional equations are

$$G''' + 8F''G + 4FG'' - 10F'G' - \phi = 4\zeta(G'^2 - GG'') \quad (11)$$

$$\phi'' + Pr(8G\theta' + 4F\phi' - G'\theta - 5F'\phi) = 4\zeta Pr(G'\phi - G\phi') + \tfrac{5}{4}\zeta^{-3/4}(1 - 5\zeta^{1/4}\eta)\exp(-5\zeta^{1/4}\eta) \quad (12)$$

and their boundary conditions are derived from eq. (9) which yields

$$\begin{aligned} &G(\zeta, 0) = G'(\zeta, 0) = \phi'(\zeta, 0) - \tfrac{5}{4}N_c\zeta^{-3/4}\theta(\zeta, 0) - 5N_c\zeta^{1/4}\phi(\zeta, 0) = 0 \\ &G'(\zeta, \infty) = \phi(\zeta, \infty) = 0 \end{aligned} \quad (13)$$

Numerical solution was carried out for the governing differential equations (7), (8), (11), and (12) together with their boundary conditions given by eqs. (9) and (13). The Nachtsheim–Swigert iterative technique (**11**) was used for computations with iterations stopped for a relative difference between two successive iterations of 0·001. The calculations were carried out for $Pr = 6{\cdot}5$ (of water at about 25°C), for ζ ranging between 0·0001 to 10 and for values of N_c up to 10. Wide ranges of ζ and N_c were chosen to cover a wide range of the absorption coefficient 'a' which depends on both the type of the absorbing fluid and its degree of contamination or colouring. The choice of these wide ranges was decided after examining the measurements of Dorsey for the absorption of radiation in water layers of different thickness as quoted by Cooper (**12**).

Beer's law, namely

$$q''_{abs} = q''(1 - e^{-ay}) \quad (14)$$

was applied to analyse Dorsey's data. The estimated value of a ranges from 6 m^{-1} to 151·5 m^{-1} for distilled water of thickness from 10 cm to 1 mm, respectively. The large variation in the value of a may be attributed to either the inaccuracy of Dorsey's measurements as stated by Cooper, or to the reflectance and spectral dependence of the absorption coefficient of distilled water, which is not considered by Beer's law. However, in the present case an absorption coefficient between $200\ m^{-1}$ and $2000\ m^{-1}$ is assumed for a blackened 'grey' water. Therefore, at this range of a and at radiation flux intensity $q'' = 900\ W/m^2$ (the average value of q'' in Saudi Arabia), the value of $x = 1$ m corresponds to $\zeta = 0{\cdot}001$ when $a = 200$ or $\zeta = 1{\cdot}74$ when $a = 2000$. This large variation in ζ with changing a for the same distance x suggests solutions for the wide range of ζ given above. In a similar way the range of N_c can be interpreted where a value of $N_c = 0{\cdot}2$ corresponds to an absorbing fluid of $a = 200\ m^{-1}$ and surrounding air moving at 5 m/s. However, a large value of $N_c = 10$ was chosen to allow for high wind velocities and/or low absorption coefficient.

RESULTS

As stated before, the solution is non-similar due to both the heat source term in the energy equation and the temperature boundary condition at the plate. Figure 2 shows the profiles of the dimensionless longitudinal velocity component F' at three different values of ζ for the same loss coefficient $N_c = 0{\cdot}5$. The values of F' tend to

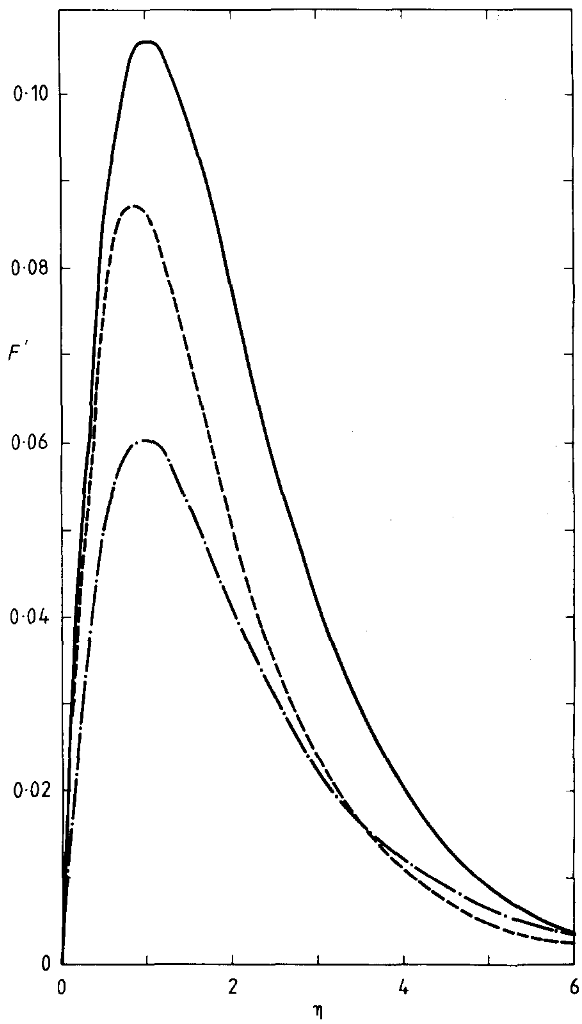

Fig. 2. Distribution of the dimensionless longitudinal velocity, F' against η at different values of ζ and at $N_c = 0{\cdot}5$. —— ζ0·001, - - - - ζ0·1, —·— ζ10

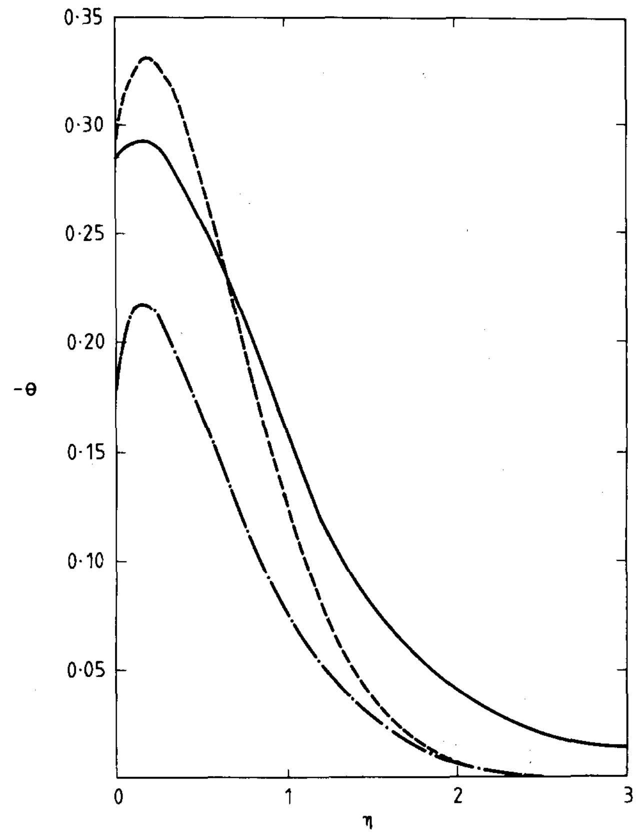

Fig. 3. Distribution of the dimensionless temperature, θ, against η at different values of ζ and at $N_c = 0{\cdot}5$, —— ζ0·001, - - - - ζ0·1, —·— ζ10

be higher and more spread at low values of ζ than at large values of ζ. The spreading of the profile can be interpreted in terms of the relative thickness of the absorbing layer to the boundary layer. The thickness of the absorbing layer (defined as the thickness at which 0·99 of the incident radiation is absorbed by the absorbing fluid) is constant as appears from Beer's law given by eq. (14). The thickness of this absorbing layer when expressed in terms of η increases with the decrease of ζ as can be inferred from eq. (4) and (5). This explains why the velocity distribution gets wide by decreasing ζ.

The profiles of dimensionless temperature θ are depicted in Fig. 3 for $N_c = 0{\cdot}5$ and at three different values of ζ. It is also noticeable that the curves are spread to relatively high values of η at small ζ when compared to the distribution at large ζ. The maximum value of θ occurs inside the fluid and not on the plate. This is because the incident radiation is absorbed by the absorbing fluid and the ideally transparent plate can only be heated through the absorbing fluid. However, this is believed to encourage solar collection by direct absorption where the losses will be reduced as a result of the low temperature at the plate.

The velocity and temperature profiles are plotted versus the distance y in Figs. 4 and 5. The magnitudes of temperature, velocity, and boundary layer thickness increase with propagation in the streamwise direction. The situation, though expected, is not clear from the profiles of F' and θ, which intersect with each other.

The effect of the loss coefficient N_c on the velocity profiles is shown in Fig. 6. An increase in the loss

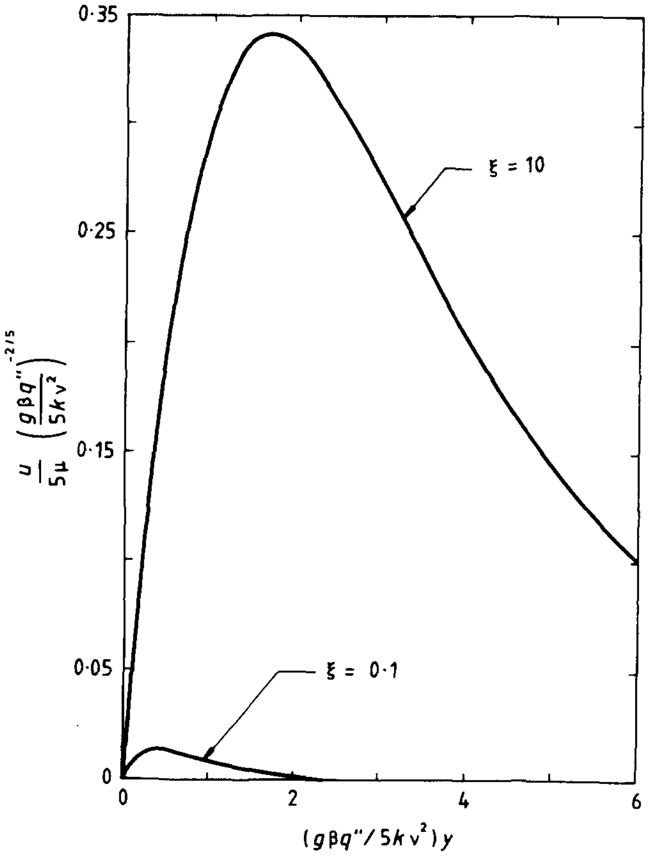

Fig. 4. Distribution of the longitudinal velocity component at different values of ζ and at $N_c = 0{\cdot}5$

coefficient decreases the value of the tangential velocity. This trend is expected since the increase in the heat loss from the plate decreases the amount of heat absorbed per unit volume of the fluid, thus decreasing the kinetic energy and thereafter the velocity.

A similar trend for the dimensionless temperature θ is shown by Fig. 7 where high values of N_c decrease the value of θ. It is also noticeable from Figs. 6 and 7 that the solution approaches an asymptotic solution for high values of N_c as predicted from curves for $N_c = 5$ and $N_c = 10$.

Referring to Fig. 1, the useful heat gain q''_u collected by the absorbing fluid is given by

$$q''_u = q'' - U(T_p - T_\infty) \tag{15}$$

A new definition of the local heat transfer coefficient is introduced based on the maximum local temperature rise, where

$$q''_u = h(T_{max} - T_\infty) \tag{16}$$

Hence

$$h = \frac{q''}{T_{max} - T_\infty} - U\frac{T_p - T_\infty}{T_{max} - T_\infty} \tag{17}$$

Dividing eq. (17) by ka and by using the definitions of θ and N_c given by eqs. (6) and (10) and the relations of eqs. (4) and (5) give

$$Nu_a = -\left(\frac{1}{5\zeta^{1/4}\theta_{max}} + N_c\frac{\theta_p}{\theta_{max}}\right) \tag{18}$$

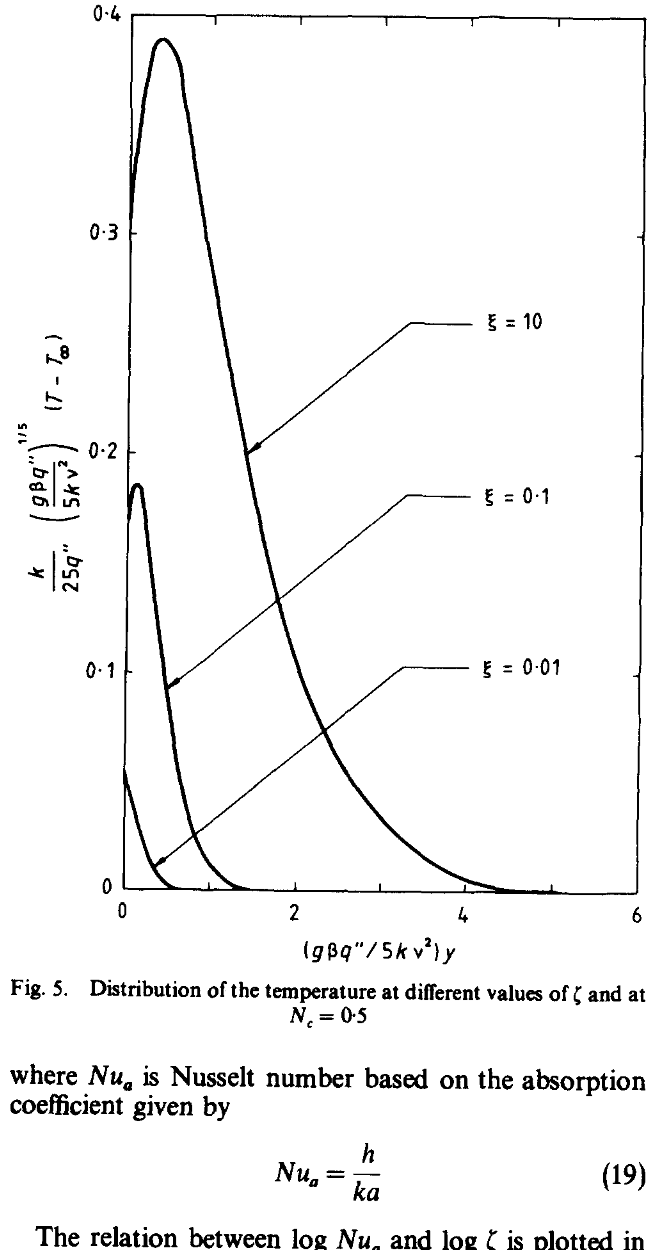

Fig. 5. Distribution of the temperature at different values of ζ and at $N_c = 0{\cdot}5$

where Nu_a is Nusselt number based on the absorption coefficient given by

$$Nu_a = \frac{h}{ka} \tag{19}$$

The relation between log Nu_a and log ζ is plotted in Fig. 8. In this figure a straight line relation is obtained between Nu_a and log ζ; this suggests the following equation for Nu_a

$$Nu_a = A\zeta^B \tag{20}$$

where A and B are functions of N_c as illustrated by the following tabulation

N_c	A	B
0	0·416	−0·25
0·5	0·275	−0·29
5, 10	0·25	−0·4

CONCLUSIONS

The problem of natural convection over a non-reflecting, non-absorbing, ideally radiation-transmitting vertical flat plate due to absorption of radiant energy in an absorbing fluid is considered. Being heated by the absorbing fluid, the plate loses heat from its external side

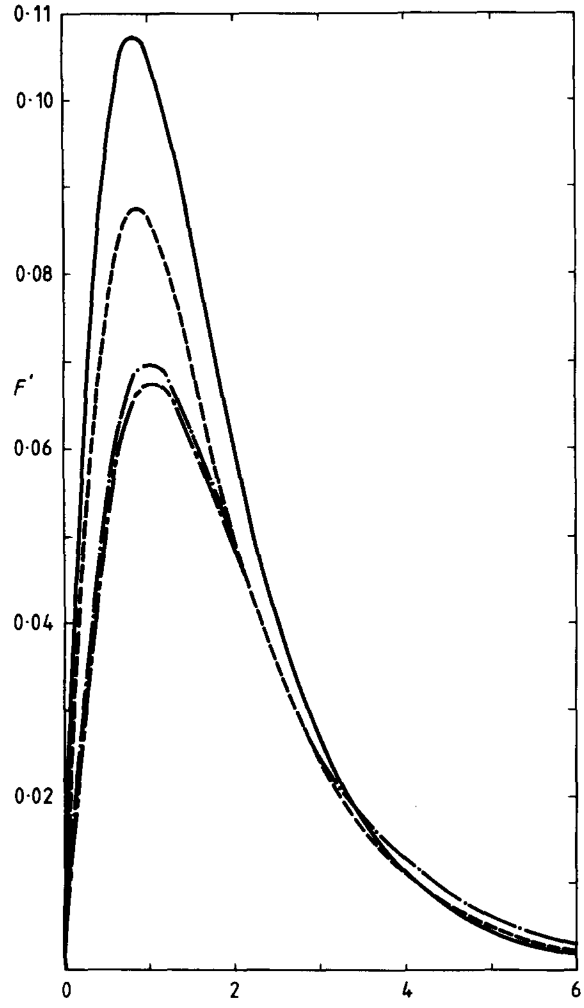

Fig. 6. Effect of the dimensionless loss coefficient N_c on the dimensionless longitudinal velocity F' at $\zeta = 0{\cdot}1$. —— $N_c 0$, ---- $N_c 0{\cdot}5$, —·— $N_c 5$, – – – $N_c 10$

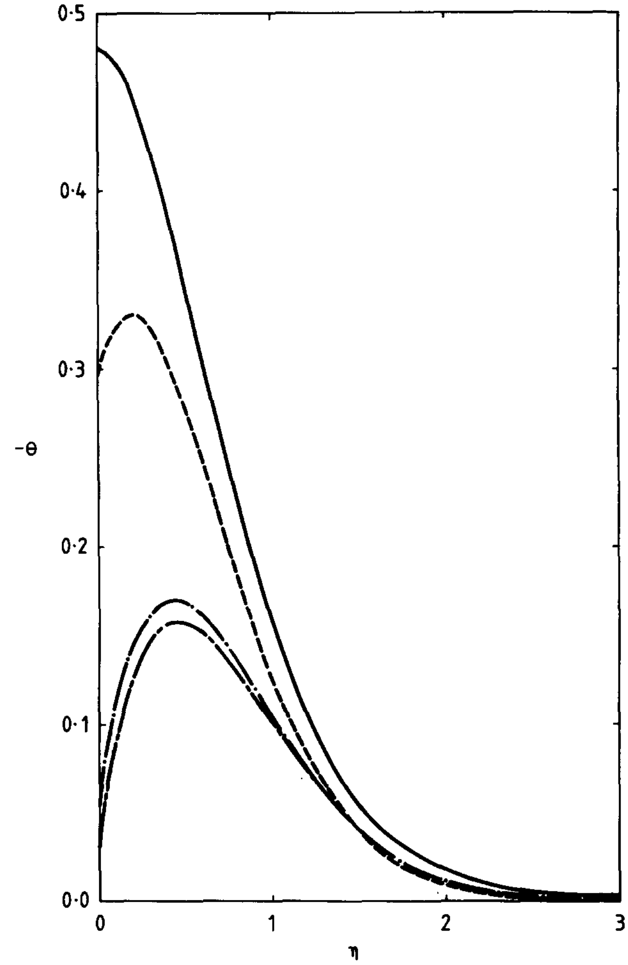

Fig. 7. Effect of the dimensionless loss coefficient N_c on the dimensionless temperature, θ, at $\zeta = 0{\cdot}1$, —— $N_c 0{\cdot}0$, ---- $N_c 0{\cdot}5$, —·— $N_c 5$, – – – $N_c 10$

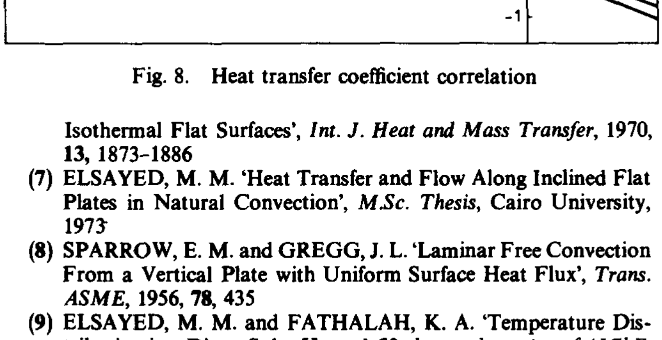

Fig. 8. Heat transfer coefficient correlation

to the surroundings. Non-similar solutions are obtained using a local non-similar technique. The results showed that the maximum temperature is found to occur inside the fluid rather than on the plate. A new definition of the heat transfer coefficient is introduced which is based on the local maximum temperature rise. The heat transfer coefficient is correlated in terms of the dimensionless streamwise distance ζ at different values of the loss coefficient N_c.

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